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The interaction between task design and technology design in creating tasks with Cabri Elem

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Both the design of tasks and the design of technology have been identified as important factors in the effective use of technology-based tasks in the classroom. By analyzing both the design of a sequence of tasks (based on didactical principles from Brousseau's (1998) theory of situations) and the affordances of Cabri Elem software it will be shown that technology can be designed in such a way as to enhance the implementation of didactical principles.

Keywords: Task design, technology, instrumental genesis, feedback, didactical variable

Design of the technology and design of the task

The potential for the contribution of Cabri II Plus and Cabri 3D to student learning is well known. However, Laborde and Laborde (2011) note that there is a gap between situations used in research and everyday teaching practice and suggest the provision of ready-made resources to help teachers take more advantage of dynamic mathematics environments. Sinclair (2003) studied pre-constructed files in a dynamic geometry environment and concluded that the design of the files was critical to their potential to support or impede student learning. There is an increasing concern about the design of technology itself. "*Design detail counts*" states Jackiw (Butler et al., 2009, p. 431). Research on resources and the problematic of quality indicates the importance of the question (Gueudet *et al*. 2011; Trgalova *et al*. 2011). Mackrell (2011) has shown that particular dynamic geometry technologies may have unclear interfaces, a lack of affordances and poor mathematical representations, all of which will hamper the use of such technology as an environment for the design of resources.

The above concerns, together with the perceived need to create a version of Cabri that was suitable to the needs of primary students has led to the development of the Cabri Elem technology. A key difference with earlier versions of Cabri is that the focus of the technology is on creating an environment for task design, acknowledging the importance of both the design of the technology itself and of the design of tasks using the technology.

Tasks in Cabri Elem are presented in "activity books" which consist of a succession of pages incorporating a sequence of tasks. Once an activity book has been created in the Cabri Elem Creator task design environment, it can be used directly by teachers and students in the more restricted environment of Cabri Elem Player.

In this paper we will analyze a particular Cabri Elem activity book in order to illustrate the interconnections between the affordances of the technology and the ability to implement particular didactic principles.

Part I. Theories of Task and Software Design

The theory of didactical situations (Brousseau, 1998) offers a theoretical framework and a number of conceptual tools to study and also to design tasks. In this theory, knowledge is a property of a system constituted by a subject and a "milieu" in interaction. Learning occurs through this interaction: the subject acts within and receives feedback from the milieu. But there is an added requirement. The signification of the knowledge that can be constructed in the interaction depends on the existence of a space of uncertainty and freedom for the subject about appropriate action and strategy. If the student has no choice, the learning outcome may have no mathematical meaning. Together with criteria for success or failure, the goal, whether teacher or student determined, is made clear, unlike in the common dynamic geometry task "drag this point and observe" where the student has no choice of action and is uncertain about what is relevant to observe. In the case of mathematical learning with technology, the relation between the technology and the milieu is complex. The milieu cannot be reduced to the technology. Technology may be one component of the milieu, (Laborde and Capponi 1994) but only the part of technology relevant to the mathematics concerned (Brousseau 1998). The same technology will also not constitute the same milieu for every subject. The milieu related to a student changes as student knowledge, both technical and mathematical, develops.

The mathematical problem and the task are key elements of a didactical situation. In the context of this contribution, we characterize a "task" by the following requirements:

- a task involves learning objectives; when a teacher proposes a task to a student, he assumes that achieving the task will cause learning;
- it involves the student encountering a mathematical problem;
- it is performed by concrete and conceptual student actions;
- it corresponds to phases of the didactical situation (in the sense of Brousseau) and is related to different values of a set of didactical variables.

Didactical variables are parameters of the situation, with values that affect solution strategies. The effects can be of three kinds: (i) a change in the validity of a strategy, where a strategy that produces a correct answer with a certain value of a didactical variable will produce an incorrect answer with another value, (ii) a change in the cost of the strategy (for example counting elements one by one is efficient for a small number but much more costly for a larger number) (iii) the impossibility of using the strategy. A combination of the different didactical variable values contributes to the task definition. The learning situation is a choice of different tasks that lead the students to construct the appropriate strategy. Thus task design will consist, for a part, in identifying the didactical variables of the situation and then choosing the succession of appropriate combinations of didactical variable values. This will be one of the foci in describing the task design process.

When creating a learning task in a computer environment, the author has to create all the elements the student will deal with: the objects the student will manipulate, the possibilities of actions on these objects and the feedback provided by the environment. The elements chosen will determine the possible milieu and the potential for learning. The feedback given by the milieu will be a second focus in describing the task design process.

Instrumental genesis involves the processes of instrumentation whereby a person builds personal utilization schemes for an artefact and instrumentalization whereby a person adapts an artefact to their own purposes, with the result that an artefact becomes an instrument to be used in the pursuit of a goal (Rabardel and Bourmaud, 2003). Instrumentation is clearly relevant to task design using tools, as it has been found to be problematic (Trouche, 2005). However, instrumental genesis is also highly relevant to the design of the technology itself. In the human computer interaction literature, "design is perhaps the most common issue addressed within the approach" (Kaptelinin and Nardi, 2006, p. 110):

> "artefacts should be designed to enable efficient transformation into instruments. [...] the importance of designing flexible, open artefacts that can be modified by users and adjusted for various tasks, including unanticipated tasks and the need for designers to take into account the actual transformation of practices and the real needs of users over the course of appropriating an artefact".

The most important design principle in the development of Cabri software is that of direct manipulation (Laborde and Laborde, 2011), which involves both action and feedback on action. This is a fundamental concept of human computer interaction, developed in the 1980's when the current almost-ubiquitous graphical user interfaces were replacing interfaces requiring text commands (Kaptelinin and Nardi, 2006, p. 82). The general principles below (Shneiderman and Plaisant, 2010, p. 196) are illustrated by examples from dynamic geometry.

1. Continuous representations of the objects and actions of interest with meaningful visual metaphors. Geometric objects and tools.

2. *Physical actions, or presses of buttons, instead of complex syntax*. Dragging to move objects, constructing a custom tool or macro.

3. Rapid, incremental, reversible actions whose effects on the objects of interest are visible immediately. The response of any geometric object when one of the points on which it is dependent is dragged.

Some of the benefits claimed for systems designed using these principles are that novices can learn basic functionality quickly, users can immediately see whether their actions are furthering their goals, users experience less anxiety and gain a sense of confidence and mastery (Shneiderman and Plaisant, 2010, p. 196) This has obvious implications for instrumentation.

However, Rabardel (2002, p. 148) suggests that direct manipulation may not always be the most appropriate means of interaction with an artefact in a learning situation. The aim, rather than making the action easier, may be to construct constraints that lead the subject to use and elaborate cognitive constructions of knowledge (p. 148). An implication is that the task designer in a technological environment must have the facility to vary the type of action required and response received as appropriate.

An important additional principle for mathematics educational software is "epistemic fidelity" in which the representations of mathematical objects must avoid any contradictions with the abstract objects they represent, both in appearance and in behaviour when manipulated (Laborde and Laborde, 2011). Rabardel (2002, p. 160) also warns that while instruments may be designed to favour the construction and manipulation of conceptualizations these impose a "world view" on users. This has consequences for design at many levels, ranging from the choice of perspective (to ensure that a cube does not appear to "pulsate" when moved) in Cabri 3D to the importance of distinguishing geometric labels and variables (Mackrell, 2011). Laborde and Laborde also feel that physical action is preferable to pressing buttons: that by directly acting on the representation of mathematical objects the user eventually perceives the abstract object itself.

Part II. Analysis of a Cabri Elem activity book

In what follows, we will look at an activity book and discuss the task design decisions and the technology affordances, which enable these decisions. The "Target" activity book addresses the French primary school level CE1 (7 year old students) and deals with the representation of numbers using place value notation. The idea arose from comparing counters on a scoreboard, where the value of the counter depends on its position on the board, with the way that the value of a digit depends on its position in a written number. It was designed by a team of ten researchers (including two of the authors of this paper), teacher educators and teachers involved in a French national project[10] whose purpose is to create resources for the teaching of mathematics in kindergarten and primary school.

Cabri Elem has the affordances of earlier Cabri technology for direct manipulation and feedback involving geometrical objects and numbers. It has been extended to more effectively model the real world with new objects (such as 3D models) and new tools (such as a realistic compass). It also enables a 3D view, as shown below.

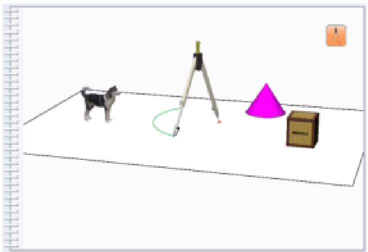

Figure 1. 3D objects and use of the realistic compass tool in Cabri Elem

The affordances for interaction with objects have also been extended, both by enabling restrictions on default behaviours (objects may be locked to prevent changes, or feedback on student actions may be delayed), and by enabling new possibilities for action, such as feedback given at the click of a button.

A key feature of Cabri Elem for task design is that the task author is entirely responsible for the student interface, as the Cabri Elem Player interface is empty at the beginning of the design. Instrumentation and student focus can be better managed by controlling the complexity of the interface, and the author is free to elaborate the milieu by choosing appropriate objects, possible actions and resultant feedback according to the aims of the task. In our example, the objects are essentially the scoreboard with three different regions, the counters, the target number and the score, as shown below.

[10] The « Mallette » project is supported by the French Ministry of Education and conducted in collaboration between the IFE Institut Français de l'Education and the COPIRELEM Commission of IREM http://educmath.ens-lyon.fr/Educmath/recherche/equipes-associees/mallette/.

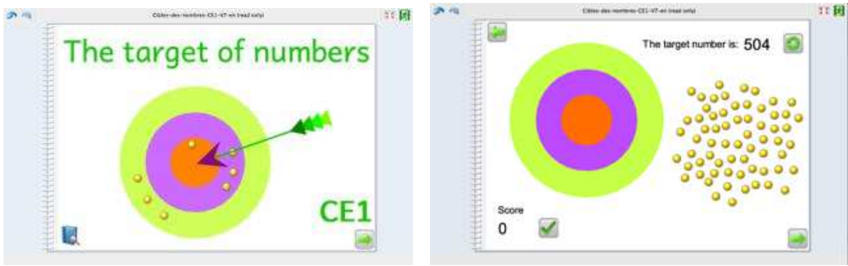

Figure 2. Title page and a task page from the "Target" activity book

The actions on the objects are simple: dragging the counters, clicking on a button to get a new target number and reset position of the counters and clicking on a button to get an evaluation.

Another key feature of Cabri Elem is the ability to include multiple pages. This enables activities to be planned as a sequence of tasks, in which an evolution in student strategies may be provoked through changes in the value of the various didactical variables. The ability to choose the way in which pages are linked also enables the provision of optional help in tool use, differentiated tasks, notes for teachers, etc.

In the task design process with Cabri Elem, didactical variables play an important role. Some are identified a priori, while others emerge during the design process as the author becomes more aware of what aspects of the situation may be changed. Once a potential variable is identified, an analysis of the ways in which this variable may be changed produces a better understanding of the possible tasks and their consequences. It also enables the creation of strategy feedback.

We will also examine three kinds of feedback. Evaluation feedback is related to the achievement of the task or part of the task. Strategy feedback aims to support the student in the course of task resolution, like scaffolding (Wood and al. 1976). It is a response to the strategy used by the student. The author needs to identify (i) configurations that are typical of a strategy and hence enable a diagnosis and (ii) new objects or actions that can be provided to help the student without changing the nature of the task. Such feedback may consist of help messages, or a graphic enlightening of contradictory elements. Another possibility is to modify the values of some didactical variables in order to make the student aware of the current strategy limitations. Direct manipulation feedback is the response of the environment to student action, and may serve the function of either of the previous types of feedback.

Page 2: a page to initiate instrumental genesis

Page 1 is a title page (Figure 2). In page 2, the main objects are presented. The student may interact with these objects, by dragging counters to different positions on the scoreboard and noticing how this affects the score. This is dynamically calculated: one, ten and one hundred for each counter in the green outside region, the purple intermediate region and the orange central region respectively. The aim of the page is to give time for instrumentation to both teachers and students. They can explore interactions with the elements that will constitute the milieu without the constraints of a particular task. It also contains a reset button which, when clicked, replaces counters in their initial positions, and a button which allows students to move on to the next page.

The changing score is direct manipulation feedback that shows students not only the effect of their action, but also that action on one object (moving a counter to a different region of the scoreboard) will affect another object (the score). The values of the different scoreboard regions were chosen by the authors in accordance with learning objectives: in a version of the activity book used with younger students the scoreboard regions only gave values of one or ten. The score is always displayed in some pages, but displayed only after a specific sequence of actions in other pages.

The score is generated by means of the point counter tool a new tool, which counts the number of points within geometric shapes (in this case the three regions of the scoreboard[11]). The idea for this tool was initiated by a primary teacher educator for work with early number: she wanted a means of creating and counting a collection of objects using technology. Other affordances used in this page are the ability to evaluate expressions (the total score being calculated from the number of points in each region), and the ability to reset the position of points.

Page 3: evaluation feedback

On page 3 the student is given a specific task: to reach a score equal to a target number, randomly generated between 1 and 999 (see Figure 2). Clicking on the reset button now in addition generates a new target number. Another new action is that the student may, in addition to comparing whether the score matches the scoreboard, click on a new button for evaluation feedback: a red frowning face if the answer is wrong, and a yellow smiling face if the answer is correct. In case of failure, the student can continue to drag counters and ask for a new evaluation: a new smiley will appear to the right of the previous one. It is important that new feedback is only generated at the student's request: otherwise a trial and error strategy not stemming from mathematical considerations could lead to success.

The key new software affordance used in this page is the Boolean function, which enables a comparison between the student response and the target number. When the feedback button is pressed, the value of the resultant Boolean (TRUE or FALSE) will determine which of the faces is shown.

Pages 4 to 7: suppression of a direct manipulation feedback and evolution of the task

From page 4 to 7 students are no longer given the direct manipulation feedback of seeing the score. They hence need to take into account the value of the counters in the different regions of the scoreboard to determine the score. "Score" was identified a priori as a possible didactical variable, with two values: visible or hidden. All objects may be either visible or invisible in Cabri Elem, enabling the author to control the level of direct manipulation feedback given.

In page 5, the number of counters is reduced so that, if the target number is over 27, a strategy that consists in placing counters only in the green units region will fail. A strategy which takes into account that a single counter can have another value than 1, i.e. using the inside regions of the scoreboard, is necessary. Therefore, another potential didactical variable is identified: the number of available counters, with two values, 3x9=27 and >27. In page 6, the target number is a multiple of ten, between 10 and 990. As there are enough counters to either leave the green region empty or to fill it with multiples of ten counters, a change of strategy is not necessary. In page 7, however, a single counter is fixed in the green region. Therefore, new strategies are required, involving the placement of a multiple of ten counters into the units region of

[11] The outer rings show an example of instrumentalization: as a ring is not a Cabri Elem geometrical shape, the authors have created non-convex polygons with touching sides.

the scoreboard. The "fixed counter" didactical variable is identified, with four values: no fixed counters, or fixed counters in the units, tens, or hundreds region.

Page 8 contains input boxes for the student to enter the values of a counter in each region of the scoreboard. The aim of this task is to summarize the key idea of the activity book, i.e. that the value of a counter depends on the scoreboard region.

Other pages of the activity book not devoted to student tasks

The first page is designed with the aim of attracting teachers and students to the activity book with an iconic representation of some of the main objects.

Pages 9 and 10 contain commentaries for teachers, reporting the main aspect of the task, the evolution from one page to another, possible student strategies (correct or not) and also the solution. The structure of the pages of the activity book was used to organise these notes and the didactical variable analysis helped to determine what information was useful. Trgalova et al. (2011) also points out that teachers find teacher notes of value.

Part III. Feedback from the school

The "Target" activity book was trialled in the spring of 2012 in two primary school classes: CE1 with the version presented here and CP (six year old students) with a version where the target number size was limited to 99. Teachers used the activity book as one resource for learning about place value and instrumentalised the book by printing pages to construct related paper and pencil tasks. They were enthusiastic about student engagement, mathematical reasoning and the evolution of strategies, but raised a number of issues. We present here some findings related to instrumentation by teachers and students, instrumentalization, strategies feedback and didactical variables.

It was expected that the strong metaphor between the task situation and real situations involving a scoreboard would, as well as providing a meaningful context, minimize the need for instrumentation: however, students expected that, as in the real situation, moving a counter would require tossing it in some way and were initially uncertain about how to do this using the software. Teachers also proposed that instrumentation would be enhanced by modifying page 2 to include a target number chosen either by the teacher according to the constraints of the class, or chosen by students in order to challenge each other.

Some students used the target number update not only to get a new number after finding a previous target but also, unexpectedly, to get a number they knew they were able to deal with. They were able to diagnose their level of expertise and this, important ability to the learning process must not be ignored by the environment. It is planned to modify pages to provoke problem resolution, but also to locally enable this usage. This example of students' instrumentalization of a functionality to adapt it to their level of expertise is a new, generalizable element in activity book design.

The number of available counters was not a didactical variable for most CE1 students, who used each region of the scoreboard and limited the number of counters they needed to drag. Many of them did not notice the reduced number of counters on page 5 and were surprised to apparently have to solve the same task again. However, for a few CE1 students and many of the younger CP students who used only the units region of the scoreboard and placed as many counters as the target number the number of available counters was indeed a didactical variable. The status of page 5 will hence be changed in further developments of the book. Instead of being automatically displayed to CE1 students, it will only be displayed as necessary, i.e. if the unit region is repeatedly filled with many more than 10 counters. The strategy feedback, resulting from our analysis in terms of didactic variables, will consist in reducing the number of counters and choosing a target number over 50.

Conclusion

In this paper the following elements have been considered: task design, construction of milieu in the context of a particular technology, didactical variable, feedback, and instrumental genesis for authors and students. In order to discuss tasks, we tried to show their mutual relationships. We have shown that software affordances and theories related to task design together enable the effective creation of resources for learning and their introduction into the classroom.

However, many elements remain to be controlled and articulated in the analysis of tasks and task design. This paper, drawn from the initial stages of a study, attempts to illustrate some of these and to suggest theoretical means to study the whole process. On the one hand the analysis has shown the potentialities of Cabri Elem, but on the other hand it has shown the complexity of the process: different instrumental geneses are involved, subjects' knowledge comes into play, didactical constraints are present, etc. However, we have also shown that there is not a distinct divide between software affordances, task design and classroom implementation: Cabri Elem affordances are based on theories of software design, but also come from the requests of researchers and educators. In turn, designing a task in the Cabri Elem environment enables a greater awareness of the potential of different didactical variables. Feedback from the classroom suggests ways in which the task may be improved.

The next stage in our study is to explore ways in which activity books could be usefully modified by any teacher. The Cabri Elem task design environment[12] will enable us to consider this new level of instrumental genesis.

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